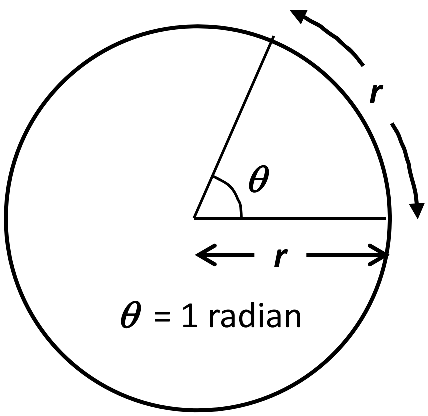
Linear and Angular Quantities

To understand the relationships between linear and angular quantities, we need to know about radians:

<u>A radian is the angle that subtends an arc length equal to the radius of the circle.</u>



A general arc has length $s = r\,\theta$ where r is the radius and θ is the angle measured in radians

Since a circle has circumference $C = 2\pi r$, this means that 2π radians = 360°.

Linear Quantity	**Angular Quantity**	**Relationship***
Displacement s	Angular Displacement θ	$s = r\,\theta$
Speed v	Angular Speed ω	$v = r\omega$
Acceleration a	Angular acceleration α	$a = r\alpha$
Mass m	Moment of inertia I	

The correspondence between linear and angular quantities gives us corresponding angular kinematic equations:

$$v_f = v_i + at \qquad \omega_f = \omega_i + \alpha t$$

$$x_f = x_i + v_i t + \frac{1}{2}at^2 \qquad \theta_f = \theta_i + \omega_i t + \frac{1}{2}\alpha t^2$$

$$v_f^2 - v_i^2 = 2a\Delta x \qquad \omega_f^2 - \omega_i^2 = 2\alpha\Delta\theta$$

*These relationships only hold if θ is measured in radians

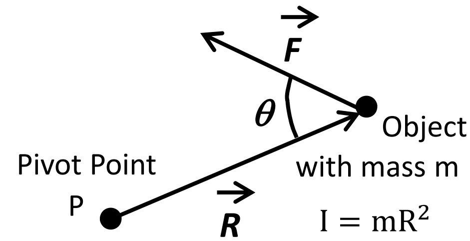
Just as the mass of an object, ***m***, determines its acceleration under a given force, $\vec{F}$, its moment of inertia, ***I***, determines its angular acceleration under a given torque, $\vec{\tau}$.

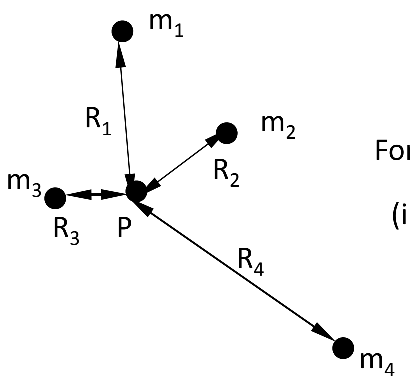
The magnitude of the torque, τ , is defined as $\tau = RF\sin\theta$

where $\vec{R}$ is the displacement from the pivot point and θ is the angle between $\vec{R}$ and $\vec{F}$.

Moment of inertia ***I*** is defined about a pivot point.

For a single object (as shown) $I = mR^2$





For a collection of N objects $I = \sum_{i=1}^{N} m_i {R_i}^2$

(in the figure N = 4)

Linear Quantity	**Angular Quantity**	**Relationship**
Mass ***m***	Moment of inertia ***I***	(see above)
Force ***F = m a***	Torque $\tau = I\,\alpha$	$\tau = F\,r\sin\theta$ *
Translational Kinetic Energy **K.E.** = $\frac{1}{2}mv^2$	Rotational Kinetic Energy **K.E.** = $\frac{1}{2}I\omega^2$	
Linear Momentum $P = mv$	Angular Momentum $L = I\omega$	

If there is no external torque on a system, then the total angular momentum is conserved (just as total linear momentum is conserved if there is no external force).

*Here θ is the angle between $\vec{F}$ and $\vec{r}$.