Enhance Student Understanding and Success in Calculus I

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Scholarship of Excellence in Teaching

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Students' Learning Problems to Solve	Some of the students • don't have the baseline knowledge necessary to understand calculus, • don't use critical thinking in math, and • don't know how to use the tools of scaffolding, • don't have a positive experience with collaboration and a good relationship with their professor.
Goals	I wanted • to increase baseline knowledge, • to emphasize the importance of critical thinking and scaffolding in math problem-solving, • students to engage with one another so that they can deepen their understanding of principles and practices, • to build good relationships with students, • to decrease the math anxiety of students, • to enhance student understanding and success in the Calculus I course.
Strategies	Apply the following pedagogical techniques more effectively • baseline knowledge, • critical thinking, • scaffolding, • collaboration.
Access the Success of the Strategies	I will make a survey asking the opinion of students about the • skill-improvement, • collaboration in problem-solving, • relationship with the professor, • learning environment, • appreciation towards math, • math anxiety,

Baseline knowledge

Critical thinking

Scaffolding

Collabo-ration

Strategies

It is essential to explain to students the importance of

- A strong **Baseline Knowledge.**

As Angela Duckworth [1] states, "Skill is not the same thing as an achievement. Without effort, your talent is nothing more than your unmet potential. Without effort, your skill is nothing more than what you could have done but didn't. With effort, talent becomes a skill, and, at the very same time, effort makes skill productive."

- **Critical thinking** *"does involve breaking down information into parts and analyzing the parts in a logical, step-by-step manner… it also involves taking other information to make a judgment or formulate innovative solutions," as The Peak Performance Center [2] summarizes.*

- **Scaffolding.** *Paula Lombardi [4] lists different facilitative tools that "can be utilized in scaffolding student learning. Among them are: breaking the task into smaller, more manageable parts; using 'think aloud or verbalizing thinking processes when completing a task; cooperative learning, which promotes teamwork and dialogue among peers; …."*

- **Collaboration.** *Elizabeth Barkley, Claire H. Major, [6], "offer that students are actively learning when they*
  - *Contribute their personal, individual perspective;*
  - *Try to understand peer's opinions;*
  - *Use higher-order learning strategies; …."*

Baseline knowledge

Critical thinking

Scaffolding

Collabo-ration

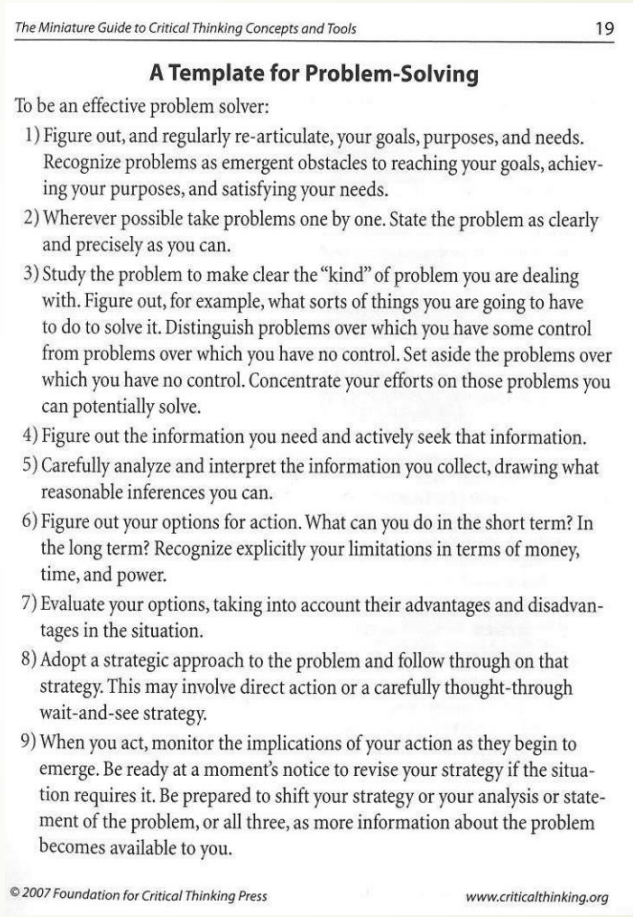
Implementation of the strategies

- **Baseline Knowledge**

I included in the homework assignments the "Essentials" assignments. These questions covered basic knowledge students should have acquired or make sure to learn. There were 4 Essentials during the semester.

- **Critical thinking**

I shared with my students the ideas in The Miniature Guide to Critical Thinking Concepts and Tools [3]. I encouraged them to make their own versions of critical thinking questions and ideas and use them whenever they are stuck with a problem. In class, solving complex questions, I referred to their ideas, demonstrating how those help in the process of problem-solving.

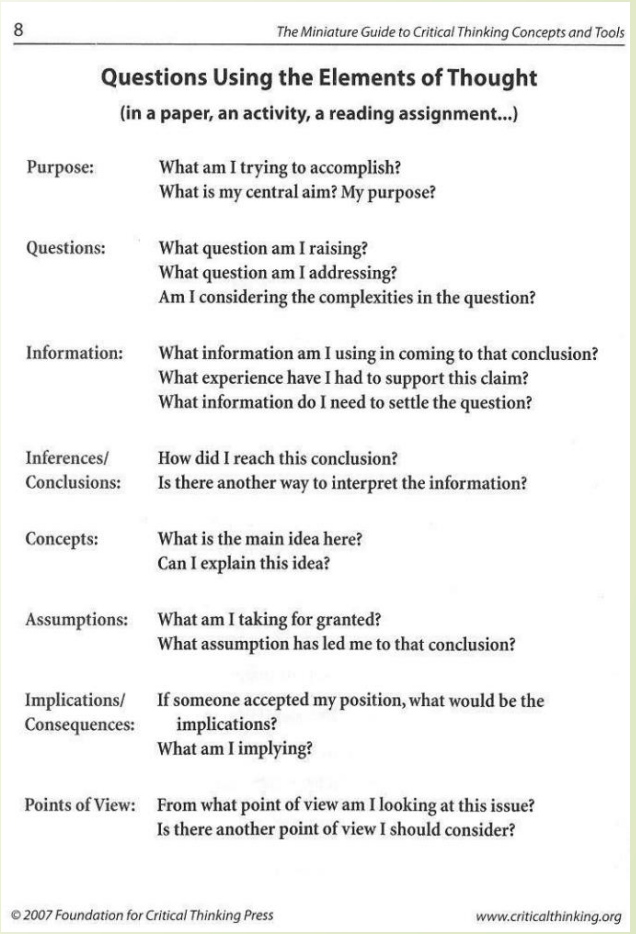


The Miniature Guide to Critical Thinking Concepts and Tools 19

A Template for Problem-Solving

To be an effective problem solver:

1) Figure out, and regularly re-articulate, your goals, purposes, and needs. Recognize problems as emergent obstacles to reaching your goals, achieving your purposes, and satisfying your needs.
2) Wherever possible take problems one by one. State the problem as clearly and precisely as you can.
3) Study the problem to make clear the "kind" of problem you are dealing with. Figure out, for example, what sorts of things you are going to have to do to solve it. Distinguish problems over which you have some control from problems over which you have no control. Set aside the problems over which you have no control. Concentrate your efforts on those problems you can potentially solve.
4) Figure out the information you need and actively seek that information.
5) Carefully analyze and interpret the information you collect, drawing what reasonable inferences you can.
6) Figure out your options for action. What can you do in the short term? In the long term? Recognize explicitly your limitations in terms of money, time, and power.
7) Evaluate your options, taking into account their advantages and disadvantages in the situation.
8) Adopt a strategic approach to the problem and follow through on that strategy. This may involve direct action or a carefully thought-through wait-and-see strategy.
9) When you act, monitor the implications of your action as they begin to emerge. Be ready at a moment's notice to revise your strategy if the situation requires it. Be prepared to shift your strategy or your analysis or statement of the problem, or all three, as more information about the problem becomes available to you.



8 The Miniature Guide to Critical Thinking Concepts and Tools

Questions Using the Elements of Thought

(in a paper, an activity, a reading assignment...)

Purpose:	What am I trying to accomplish? What is my central aim? My purpose?
Questions:	What question am I raising? What question am I addressing? Am I considering the complexities in the question?
Information:	What information am I using in coming to that conclusion? What experience have I had to support this claim? What information do I need to settle the question?
Inferences/ Conclusions:	How did I reach this conclusion? Is there another way to interpret the information?
Concepts:	What is the main idea here? Can I explain this idea?
Assumptions:	What am I taking for granted? What assumption has led me to that conclusion?
Implications/ Consequences:	If someone accepted my position, what would be the implications? What am I implying?
Points of View:	From what point of view am I looking at this issue? Is there another point of view I should consider?

Baseline knowledge

Critical thinking

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Collabo-ration

Implementation of the strategies

- **Scaffolding**

Jose Antonio Bowen [5] advises, "Create structure and scaffolding. Higher-numbered courses require more difficult thinking … Increase cognitive complexity with more self-directed work." I encouraged students to break down the problems into more manageable parts ([4]) and do step-by-step solutions. During group work, they had a chance to 'think aloud' and discuss the problem with their team members [4].

- **Collaboration**

During classes, students worked together on math problems in groups. Teamwork and, after that, the presentation of a solution by a group are very beneficial in providing positive experiences regarding the importance of individual contributions to the solution of a problem.

Students also had to accomplish a group research project with their team members during the semester. During the research project, they also experienced that everyone has an essential role in accomplishing the project [6].

Baseline knowledge

Critical thinking

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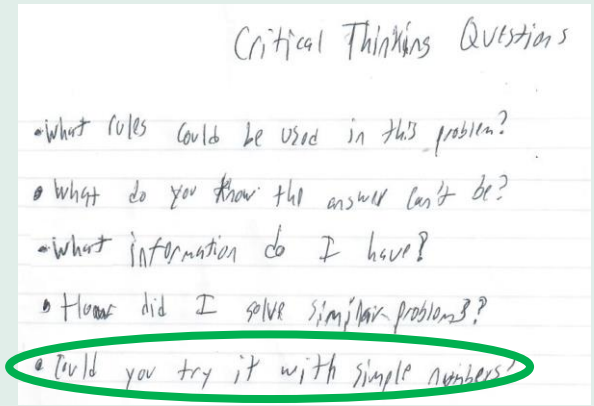
Collabo-ration

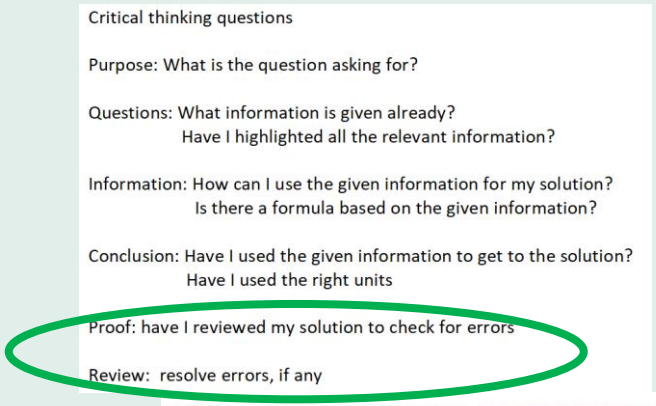
Implementation of the strategies – examples & results

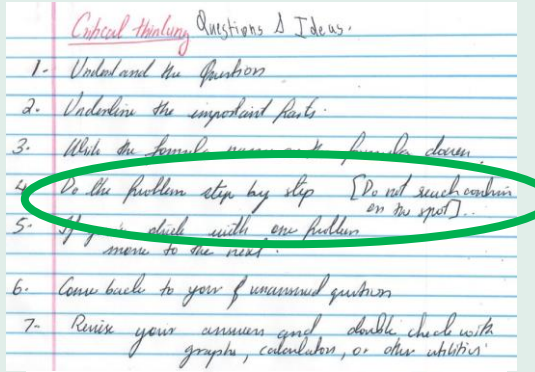
Baseline Knowledge - Results of the "Essentials" assignments

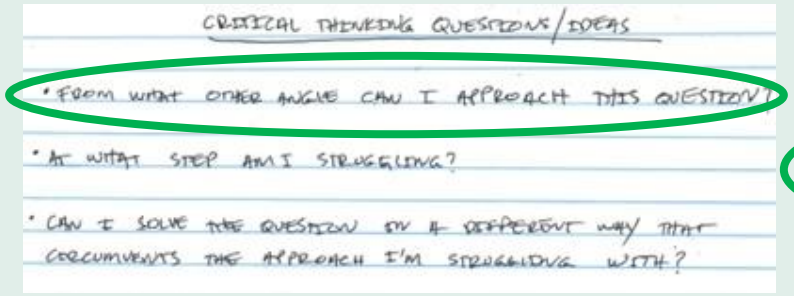
The average score of students who submitted these assignments was consistently above 94%. (Students missed only 3% of these assignments throughout the semester.)

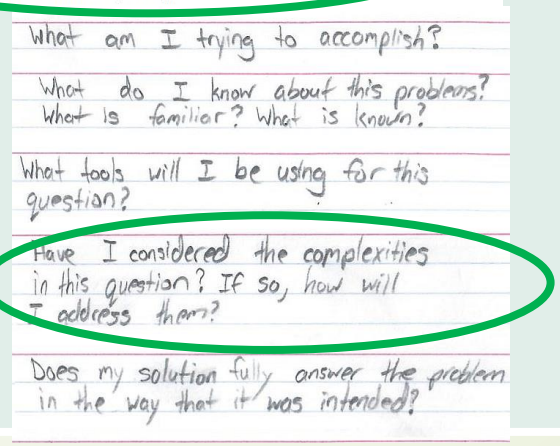
Critical thinking – students' critical thinking questions/ideas

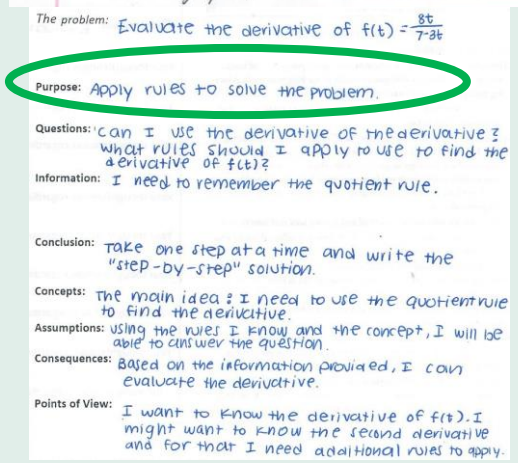












Baseline knowledge

Critical thinking

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Implementation of the strategies – examples & results

Using critical thinking questions/ideas in class

Suppose that we want to send a box as checked-in luggage on an airplane. The base of the box is a square. What is the shape of the largest volume box? (For domestic flights, the usual size limit is the following: The sum of the three edges (length, width, and height) should be less than 62 inches.)

Six Steps in Solving Optimization Problems

1. Understand the Problem.

Purpose: What is the question asking for?

What am I trying to accomplish?

What are the goals?

•What information do I have?

Questions: What information is given already?

Have I highlighted all the relevant information?

What do I know about this problem? What is familiar? What is known?

2. Draw a Diagram.

3. Introduce Notation for the Volume (V) to be Optimized.

4. Find an Expression for V.

Information: How can I use the given information for my solution?

Is there a formula based on the given information?

What tools will I be using for this question?

What information/knowledge can we use?

What is the main idea?

•What rules could be used in this problem?

using the rules I know and the concept, I will be able to answer the question.

• At what step am I struggling?

What are the steps to take?

5. Express V as a function of one variable.

6. Optimize; Find the Absolute Extrema.

7. Check your answer.

Proof: have I reviewed my solution to check for errors

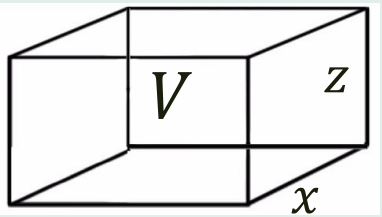
Review: resolve errors, if any

Have I considered the complexities in this question? If so, how will I address them?

Does my solution fully answer the problem in the way that it was intended?

2.

3.



4. $V = (x)(x)(z)$

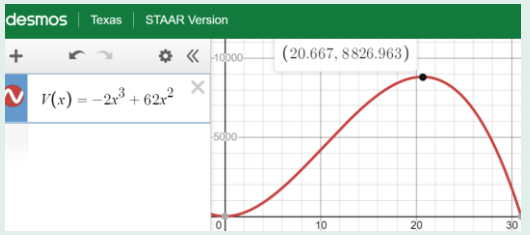
$x + x + z \leq 62\ inches$

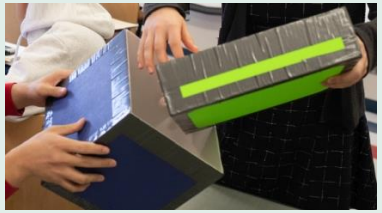
5. $x + x + z = 62$

$z = 62 - 2x$

$V(x) = x^2(62 - 2x)$

6. Using the Closed Interval Method, (see the step-by-step solution on the next slide) we find the maximum volume to be $\approx 8827\ \text{in}^3$, when $x = z = \frac{62}{3}$



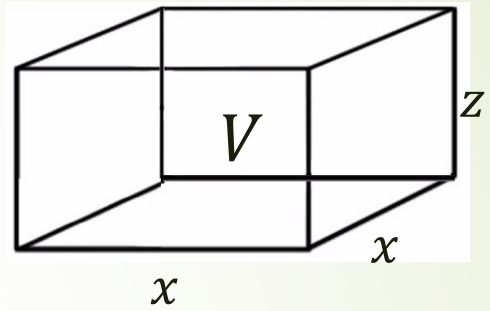


Using the Closed Interval Method to find the maximum with a step-by-step solution:

$$V(x) = x^2(62 - 2x)$$

$$V(x) = x^2(62 - 2x) = -2x^3 + 62x^2,$$

$x \geq 0$ and $62 - 2x \geq 0$;
so $0 \leq x \leq 31$



Since we are looking for extrema of a function over a closed interval, we can use the Closed Interval Method

The Closed Interval Method. To find the *absolute* maximum and minimum values of a continuous function *f* on a closed interval [0, 31]:

1. Find the values of *V* at the critical numbers of *f* in (0, 31).
2. Find the values of *V* at the endpoints of the interval.
3. The largest of the values from Steps 1 and 2 is the absolute maximum value; the smallest of these values is the absolute minimum value.

1. $V'(x) = -6x^2 + 124x,$

To find critical numbers, $V'(x) = 0$, so $x = 0$, and $x = \frac{62}{3}$.

Only $x = \frac{62}{3}$ is a critical number in (0, 31). $V\left(\frac{62}{3}\right) = \left(\frac{62}{3}\right)^3$.

2. $V(0) = 0, \qquad V(31) = 0$

3. $\max\left\{0, \left(\frac{62}{3}\right)^3\right\} = \left(\frac{62}{3}\right)^3 \approx 8827 \text{ in}^3$

The shape of the largest volume box is a cube with edges equal to $\frac{62}{3}$ inches.

Baseline knowledge

Critical thinking

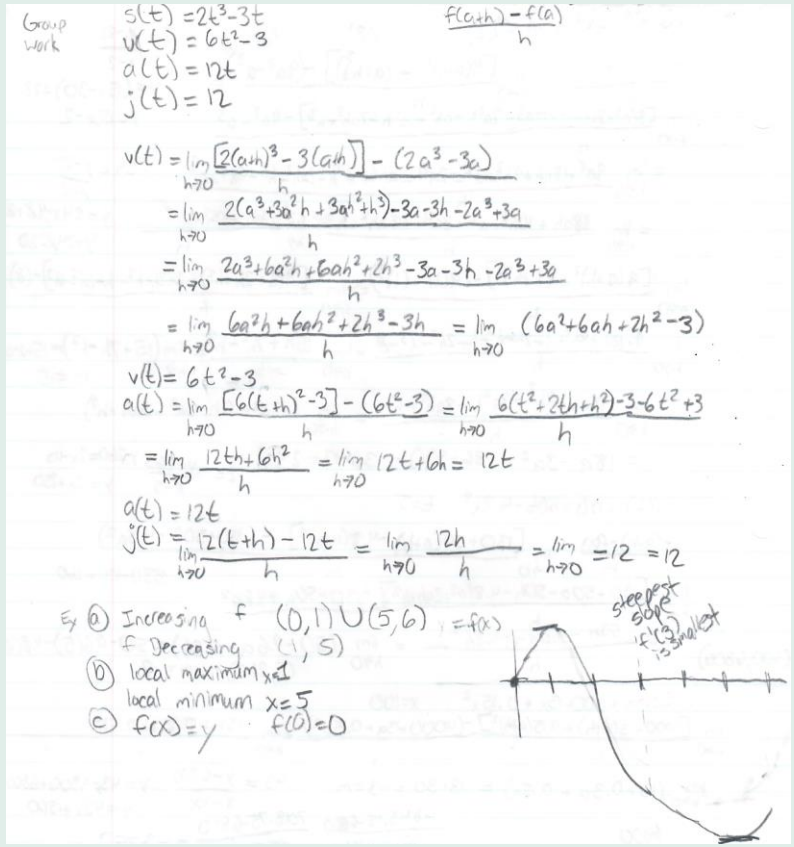
Scaffolding

Collabo-ration

Implementation of the strategies – examples & results

Scaffolding - Examples of step-by-step solutions

Group-work in class:



Solution at an exam:

12) A company estimates that the marginal cost (in dollars per item) of producing x items is $1.72 - 0.006x$.

a) If the cost of producing one item is \$540, find the cost of producing 120 items.

b) Give the equation of the cost function.

Show your work, a step-by-step solution here:

a) marginal cost = $1.72 - 0.006x = c'(x)$

$c(x) = 1.72x - \frac{0.006x^2}{2} + k$

$= 1.72x - 0.003x^2 + k$

$c(1) = 1.72(1) - 0.003(1)^2 + k = 540$

$k = 540 - 1.72 + 0.003$

$k = 538.283$

b) $c(x) = 1.72x - 0.003x^2 + 538.283$

$c(120) = 1.72(120) - (0.003)(120)^2 + 538.283$

$= 206.4 - 43.2 + 538.283$

$= 701.483$

Your answers to the questions:

The cost of producing 120 items: \$701.483
The cost function: $c(x) = 1.72x - 0.003x^2 + 538.283$

Max points: 12

Baseline knowledge

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Implementation of the strategies – examples & results

Collaboration – Examples

Group research project of a team

Group Project Topic: Cancer (specifically Monitoring Tumors)

(*1st student*) Cancer is a disease in which abnormal cells divide uncontrollably and damage body tissue. Cancer occurs when a cell develops mutations in its DNA, and the mutations allow the cell to keep on multiplying and dividing when healthy cells would normally die. …

(*2nd student*) Now, let's take a look at how calculus is used in the monitoring tumors and regarding cancer. Calculus is used in a multiple of ways. First, calculus is used to calculate the growth or shrinkage of a tumor. … Secondly, calculus can be used in calculating the total number of cells in a tumor, … Third, the progression or reversion of cancer can be found through calculus by the use of the exponential function. …

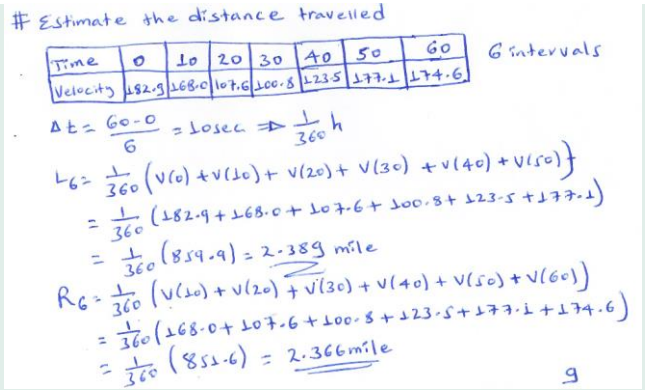
(*3rd student*) The application of calculus on tumor growth can be used to find the volume of Cancer, the growth or decay rate of the tumor, the stage when the tumor will stop growing, the treatment that should be given, and to find the critical point when the tumor becomes dangerous… There are two models of tumor growth: the Exponential growth model and the Gompertzian Model, the logistic growth (growth rate slows over time)

In Gompertz Model, by Anti-differentiating dN/dt = N [a – b * ln (N)] one can figure out the number of tumor cells in the tumor. …

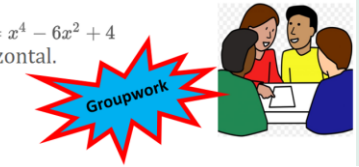
In-class group work

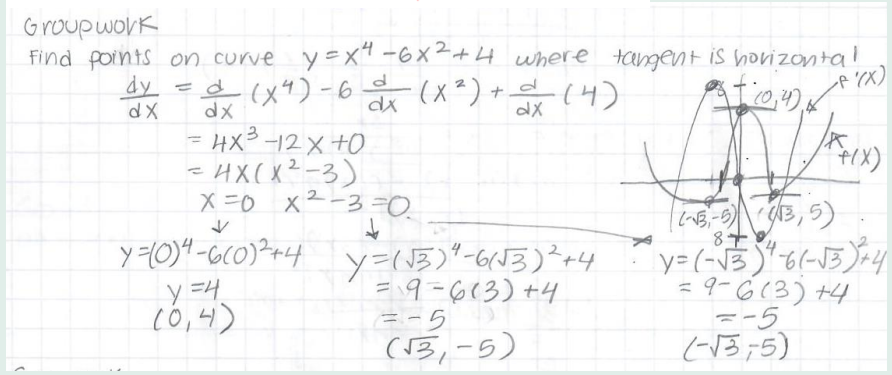
The table shows speedometer readings at 10-second intervals during a 1-minute period for a racing car.
Estimate the distance the race car traveled during this time period using the velocities
a) at the beginning of the time intervals;
b) at the end of the time periods.

Time (s)	Velocity (mi/h)
0	182.9
10	168.0
20	107.6
30	100.8
40	123.5
50	177.1
60	174.6



Find the points on the curve $y = x^4 - 6x^2 + 4$ where the tangent line is horizontal.





Baseline knowledge

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Access the Success of the Strategies

At the end of the semester, I made a survey asking Students' opinions about

1) skill-Improvement,

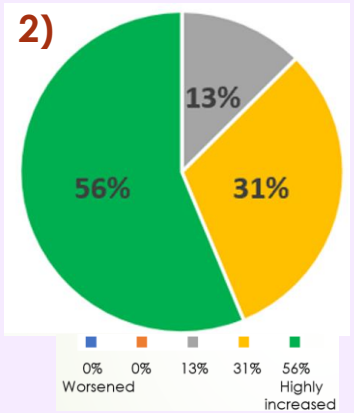
2) collaboration in problem-solving,

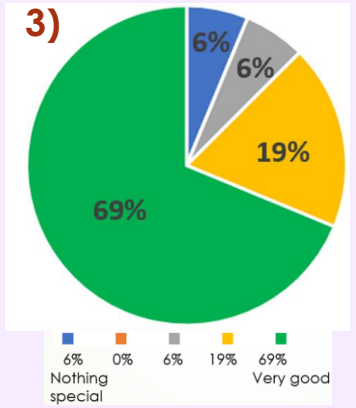
3) relationship with the professor,

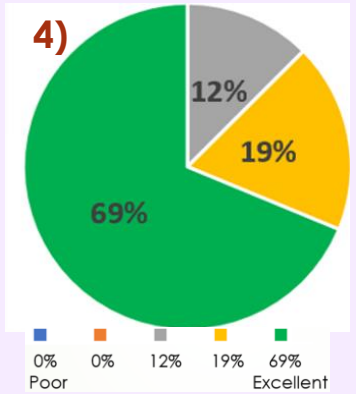
4) learning environment,

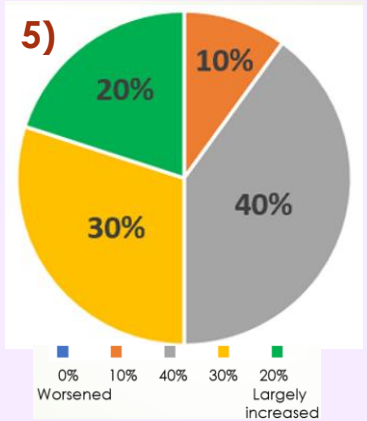
5) appreciation towards math,

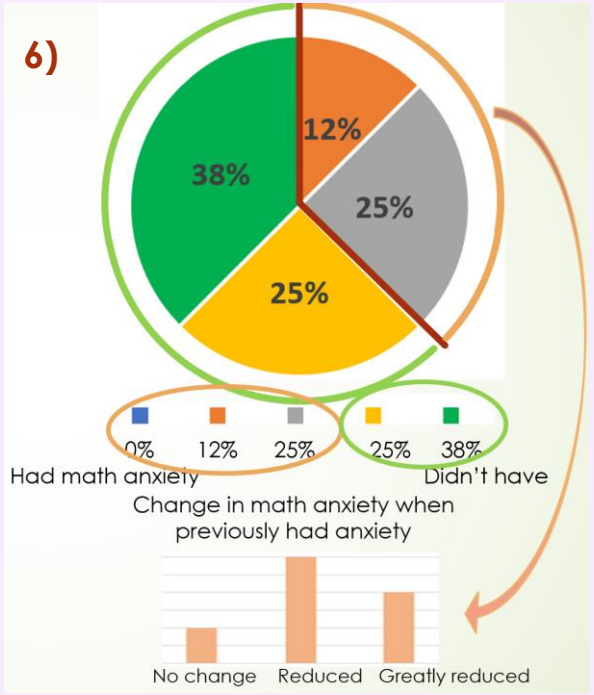
6) math anxiety











Survey

Skill-Improvement

My expectation was that these strategies positively affected students' math skills and general skills.

Regarding math skills, the average final percentage score of the class was 85%.

There are seven general skills that more than half of the students felt that this class helped them to improve.

What skills did this class help you to improve?

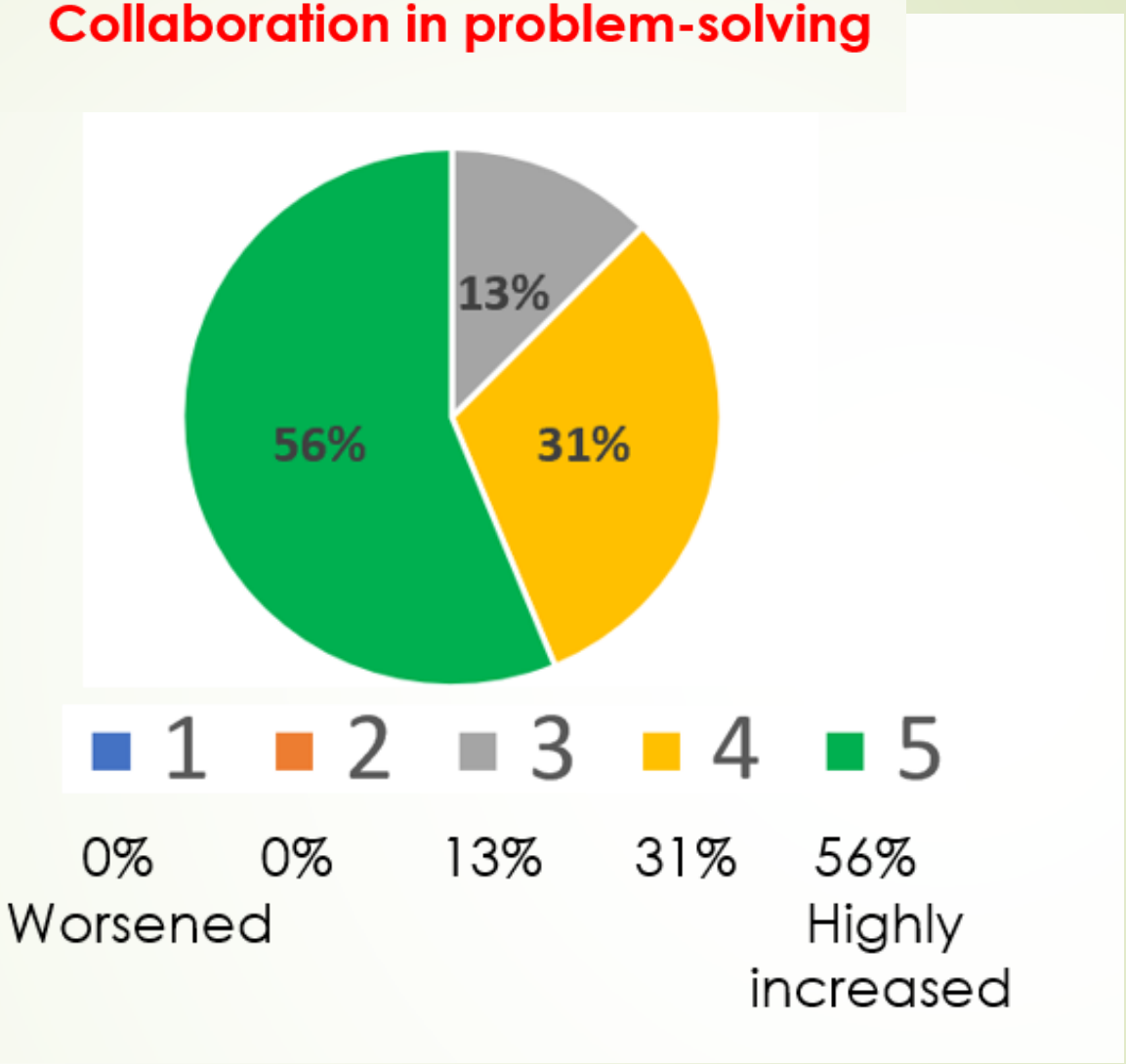
Skill-Improvement -

How did your collaboration in problem-solving change in this class?

Survey

Collaboration in problem-solving

The collaboration skills of students and their willingness to work together improved significantly. Most of my students think that collaboration is beneficial for them. One of my students told me that she appreciates that I take teamwork seriously.



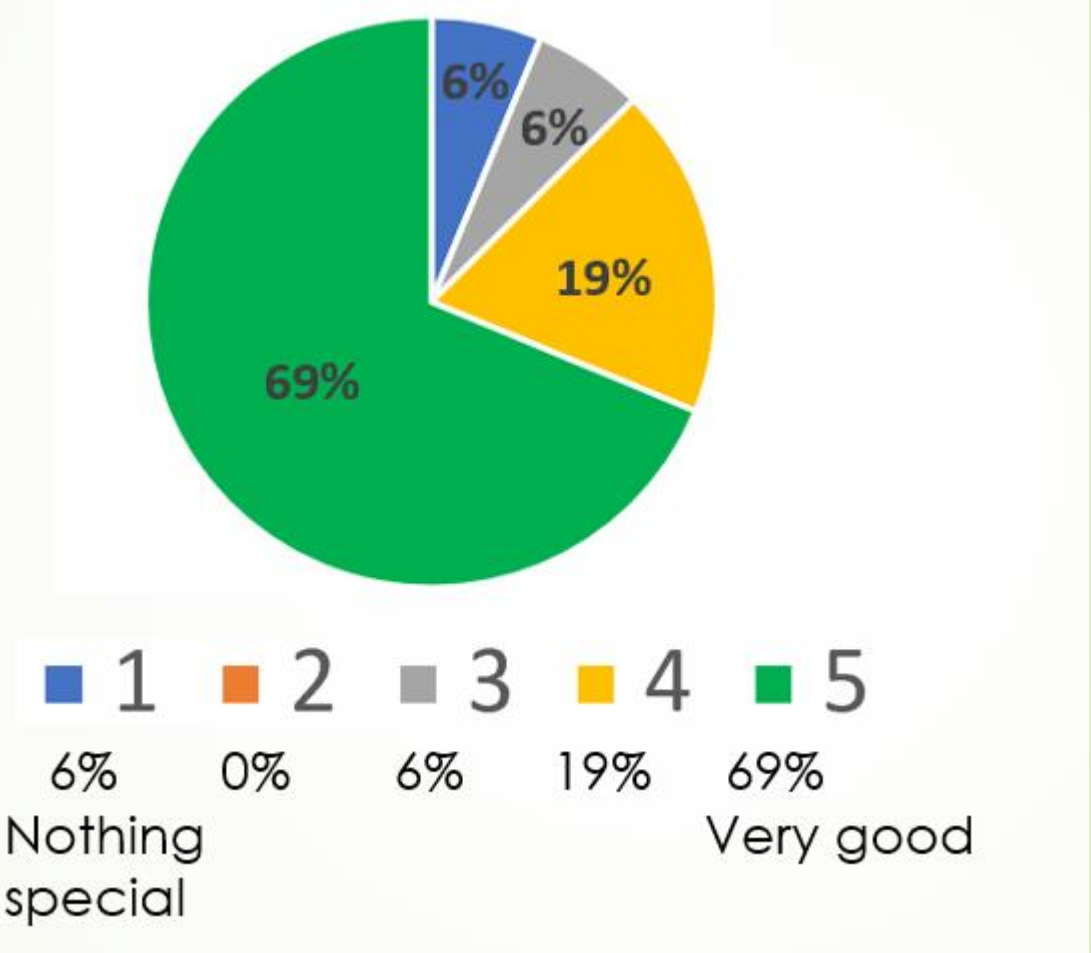
Survey

Relationship with the professor

More than 2/3 of the students expressed that their relationship with the professor was very good, and an additional 19% considered it good.

How was your relationship with the professor?

Relationship with the professor

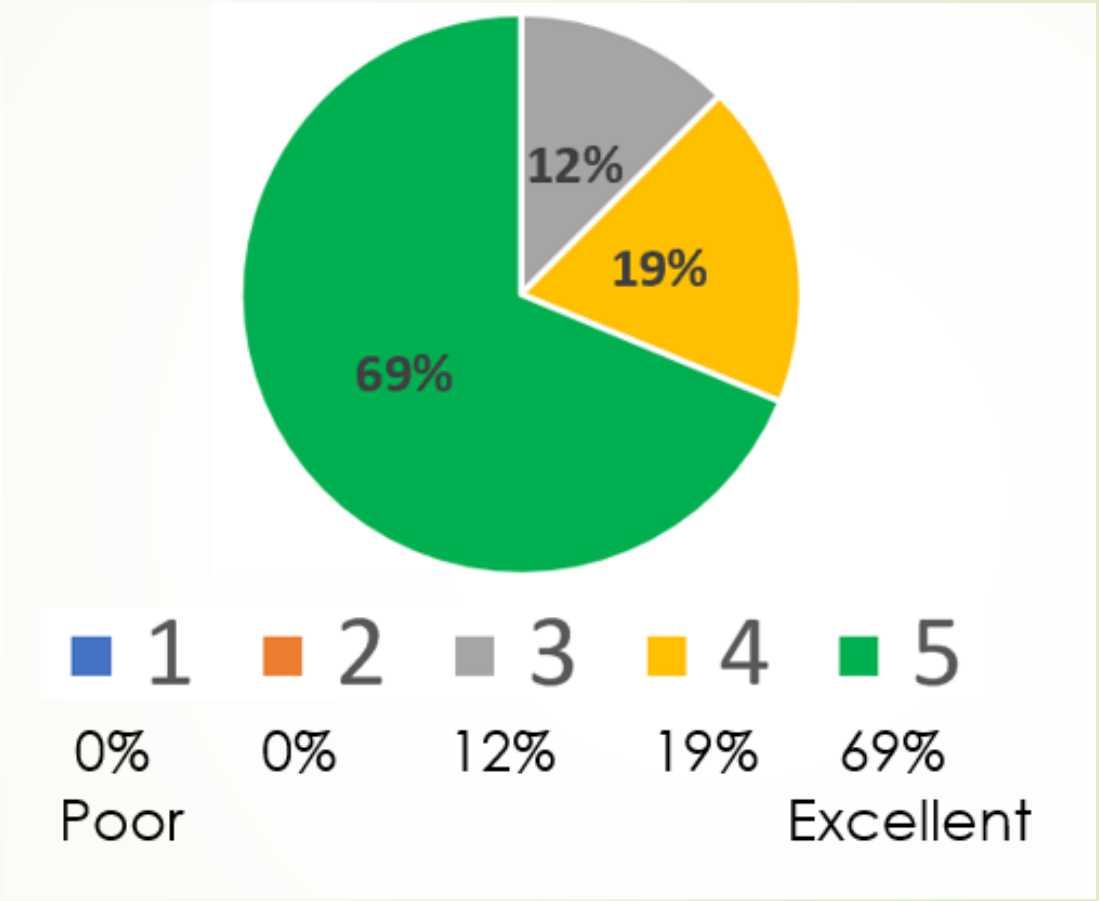


Survey

Learning environment

More than 2/3 of the students considered the learning environment excellent, and an additional 19% considered it good.

How did you like the learning environment in this class?



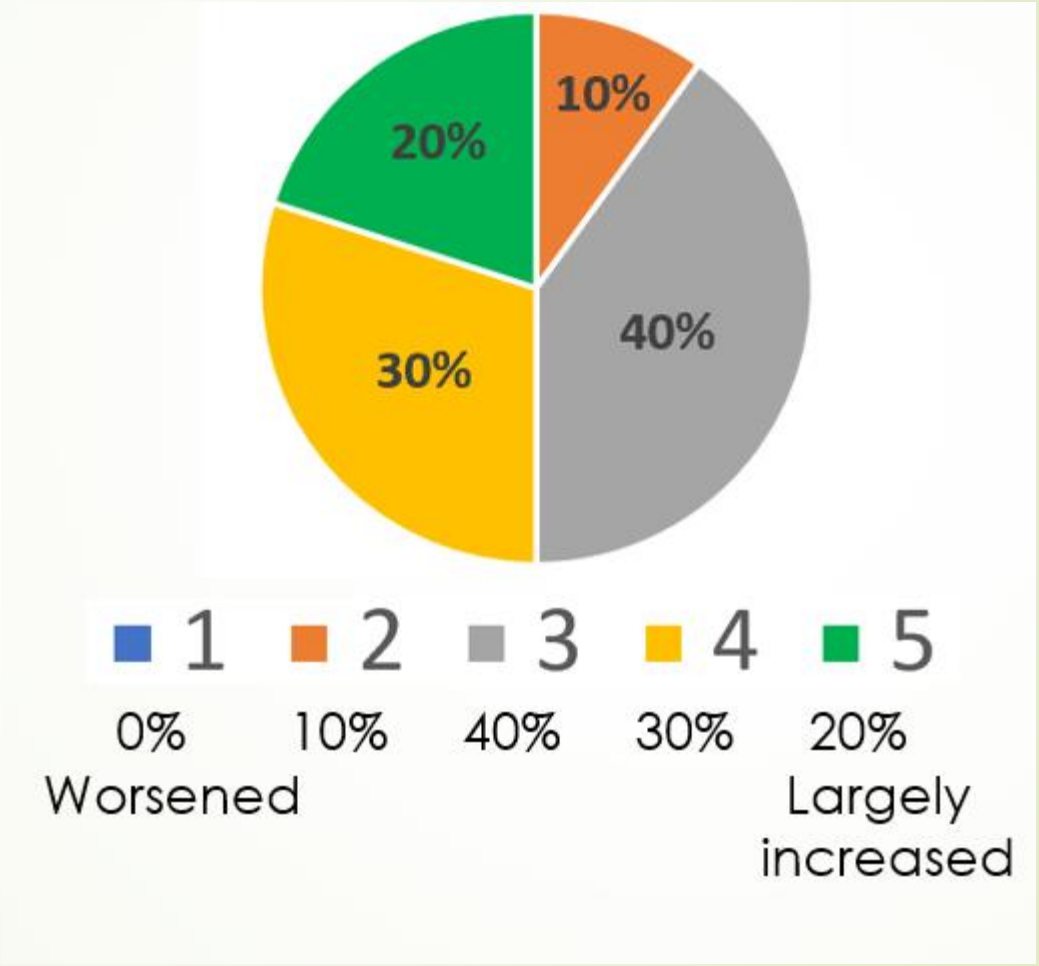
Survey

Appreciation towards math

A huge benefit of group work is that students don't feel frustrated when they work together on a math problem. They gain positive experiences while doing math, and understand that working on math problems with their peers is also fun.

Did this course have any effect on your appreciation for math?

Appreciation towards math



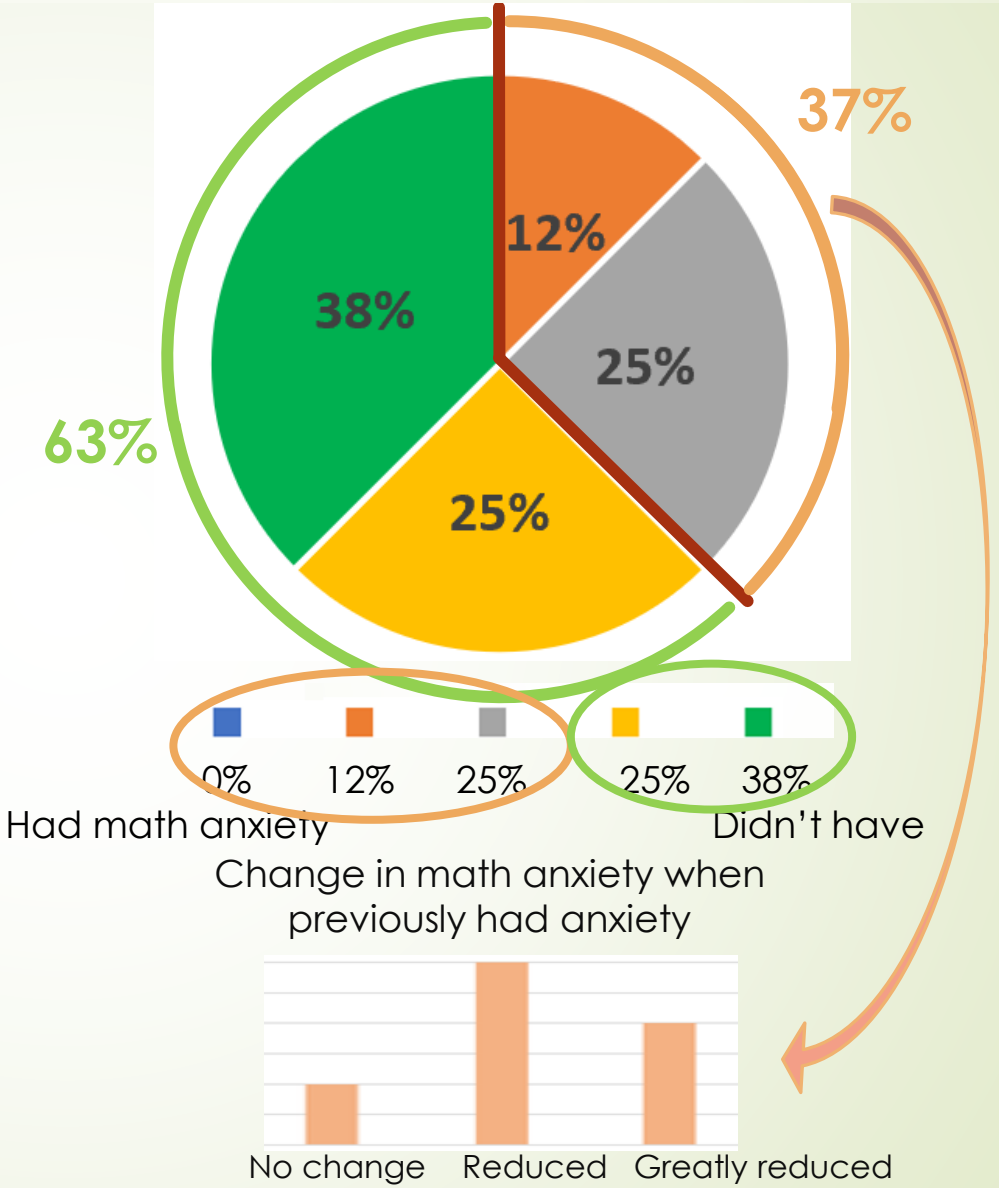
Did you have math anxiety previously? What effect did this course have on that?

Survey

Math anxiety

Students, who had no or limited anxiety, experienced some improvement or no change.

More than 80% of students who had anxiety experienced that their math anxiety was reduced.



Baseline knowledge

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Did the strategies work?

The answer is definitely yes, since

- **Baseline Knowledge**

The scores on the "Essentials" assignments intended to measure students' baseline knowledge were high, and students missed only 3% of these assignments.

- **Critical thinking**

Students who used critical thinking to solve the more difficult questions during exams achieved better results.

- **Scaffolding**

Looking through students' exams, one could observe the causation between the effective usage of step-by-step solutions and better achievements.

- **Collaboration**

Students' collaboration skills improved, and they considered collaboration beneficial.

The final average score of the class was 85%. It demonstrates that applying these strategies was very helpful in enhancing students' understanding and success.

Students' engagement was essential while using these strategies.

Baseline knowledge

Critical thinking

Scaffolding

Collabo-ration

References

Baseline Knowledge

[1] Angela Duckworth; Grit: The Power of Passion and Perseverance. “Skill is not the same thing as an achievement. Without effort, your talent is nothing more than your unmet potential. Without effort, your skill is nothing more than what you could have done but didn’t. With effort, talent becomes skill and, at the very same time, effort makes skill productive.”

Critical thinking

[2] The Peak Performance Center: “Critical thinking does involve breaking down information into parts and analyzing the parts in a logical, step-by-step manner… it also involves taking other information to make a judgment or formulate innovative solitons.” https://thepeakperformancecenter.com/educational-learning/thinking/critical-thinking/

[3] The Miniature Guide to Critical Thinking by John Paul and Linda Adler contains the Questions Using the Elements of Thoughts and the Template for Problem Solving. https://web.iitd.ac.in/~nkurur/2015-16/IIsem/cml522/CriticalThinking.pdf

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Scaffolding

[4] Paula Lombardi. Instructional Methods, Strategies and Technologies to Meet the Needs of All Learners: "Many different facilitative tools can be utilized in scaffolding student learning. Among them are: breaking the task into smaller, more manageable parts; using 'think aloud or verbalizing thinking processes when completing a task; cooperative learning, which promotes teamwork and dialogue among peers; …" https://granite.pressbooks.pub/teachingdiverselearners/

[5] Jose Antonio Bowen: Teaching Change: How to Develop Independent Thinkers, Using Resilience, Relationships, and Reflection. "Create structure and scaffolding. Higher-numbered courses require more difficult thinking … Increase cognitive complexity with more self-directed work."

Collaboration

[6] Elizabeth Barkley, Claire H. Major in Engaged Teaching: "we offer that students are actively learning when they

- contribute their personal, individual perspective;
- try to understand peer's opinions;
- use higher-order learning strategies; …"

[7] Collaborative Learning Techniques: A Handbook for College Faculty

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Elizabeth Lugosi

Thank you for your attention

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